

IMPROVED EXPONENTIAL RATIO CUM DUAL TO RATIO TYPE ESTIMATOR OF POPULATION MEAN

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Abstract. *In the present paper, an efficient exponential ratio cum dual to ratio type estimator has been proposed to estimate the population mean of the variable under consideration by using simple random sampling scheme. The bias and mean squared error of the proposed estimator have been discussed up to the first order of approximation. A comparison has been made with existing similar estimators obtained by prominent researchers engaged in this area of interest. An improvement has been reflected in terms of mean squared error (MSE). The numerical demonstration has been presented to gain the better insight into efficiency criterion of the estimator under study.*

Key words: *Exponential estimator, dual to ratio estimator, bias, MSE, efficiency.*

1. INTRODUCTION

The use of auxiliary information increases precision of the estimate of the parameter of variable under study. Many authors have proposed improved estimator in terms of greater precision. Cochran (1940) used auxiliary information and proposed the usual ratio estimator of population mean. Robson (1957) and Murthy (1964) worked out independently on usual product estimator of population mean. Searls (1964) and Sisodia and Dwivedi (1981) used coefficient of variation of study and auxiliary variables respectively. Srivenkataramana (1980) first time proposed the dual to ratio estimator for estimating population mean. Singh and Tailor (2005), Tailor and Sharma (2009) worked on ratio-cum-product estimators. Sharma and Tailor (2010) proposed a ratio-cum-dual to ratio estimator for the estimation of finite population mean of the study variable y . Bahl and Tuteja (1991) were the first to suggest an exponential ratio type estimator for the estimation of population mean of the variable under study using auxiliary information.

Let $U = (U_1, U_2, \dots, U_N)$ be the finite population of size N out of which a sample of size n is drawn with simple random sampling without replacement technique. Let y and x be the variable under study and the auxiliary variables respectively. Let $\bar{Y} = \frac{1}{N} \sum_{i=1}^N Y_i$ & $\bar{X} = \frac{1}{N} \sum_{i=1}^N X_i$

be the population means of study and the auxiliary

variables and $\bar{y} = \frac{1}{n} \sum_{i=1}^n y_i$ & $\bar{x} = \frac{1}{n} \sum_{i=1}^n x_i$ be the respective sample means.

Cochran (1940) proposed the classical ratio estimator for estimating population mean as

$$\bar{y}_R = \bar{y} \left(\frac{\bar{X}}{\bar{x}} \right) \quad (1.1)$$

With mean squared error

$$MSE(\bar{y}_R) = f \bar{Y}^2 [C_y^2 + C_x^2(1-2K)] \quad (1.2)$$

where

$$C_y^2 = \frac{S_y^2}{\bar{Y}^2}, \quad C_x^2 = \frac{S_x^2}{\bar{X}^2}, \quad K = \frac{C_{xy}}{C_x^2} = \rho \frac{C_y}{C_x},$$

$$S_y^2 = \frac{1}{N} \sum_{i=1}^N (Y_i - \bar{Y})^2, \quad S_x^2 = \frac{1}{N} \sum_{i=1}^N (X_i - \bar{X})^2,$$

$$S_{xy} = \frac{1}{N} \sum_{i=1}^N (Y_i - \bar{Y})(X_i - \bar{X}), \quad f = \frac{1}{n} - \frac{1}{N}$$

and ρ is the correlation coefficient between x and y .

Srivenkataramana (1980), using the transformation

$$x_i^* = \frac{(N\bar{X} - nx_i)}{(N-n)} \text{ or}$$

$$x_i^* = (1+g)\bar{X} - gx_i, \quad i=1,2,\dots,N,$$

which usually gives $\bar{x}^* = (1+g)\bar{X} - g\bar{x}$ where

$$g = \frac{n}{N-n}, \text{ obtained dual to ratio estimator as}$$

$$\bar{y}_R^{(d)} = \bar{y} \left(\frac{\bar{x}^*}{\bar{X}} \right) \quad (1.3)$$

with mean squared error

$$MSE(\bar{y}_R^{(d)}) = f \bar{Y}^2 [C_y^2 + gC_x^2(g-2K)] \quad (1.4)$$

Sharma and Tailor (2010) suggested a ratio-cum-dual to ratio estimator as

$$\hat{Y}_{bkl} = \bar{y} \left[\alpha \left(\frac{\bar{X}}{\bar{x}} \right) + (1-\alpha) \left(\frac{\bar{x}^*}{\bar{X}} \right) \right] \quad (1.5)$$

where α is a suitably chosen scalar. If $\alpha = 1$ and $\alpha = 0$, \hat{Y}_{bkl} reduces to estimators \bar{y}_R and $\bar{y}_R^{(d)}$ respectively.

The mean squared error of the estimator \hat{Y}_{bkl} is

$$MSE(\hat{Y}_{bkl}) = f \bar{Y}^2 [C_y^2 + \alpha_1 C_x^2 (\alpha_1 - 2K)] \quad (1.6)$$

Its minimum MSE is equal to usual linear regression estimator. Bahl and Tuteja (1991) proposed an exponential ratio type estimator as

$$t_1 = \bar{y} \exp\left(\frac{\bar{X} - \bar{x}}{\bar{X} + \bar{x}}\right) \quad (1.7)$$

with mean squared error as

$$MSE(t_1) = f \bar{Y}^2 [C_y^2 + C_x^2 (\frac{1}{4} - K)] \quad (1.8)$$

The exponential dual to ratio type estimator is as follows

$$t_2 = \bar{y} \exp\left(\frac{\bar{x}^* - \bar{X}}{\bar{x}^* + \bar{X}}\right) \quad (1.9)$$

where:

$$X_i^* = \frac{(N\bar{X} - nX_i)}{(N - n)} \text{ or}$$

$$X_i^* = (1 + g)\bar{X} - gX_i, \quad i = 1, 2, \dots, N, \text{ which usually}$$

gives $\bar{x}^* = (1 + g)\bar{X} - g\bar{x}$ where $g = \frac{n}{N - n}$.

The mean squared error of the estimator t_2 is

$$MSE(t_2) = f \bar{Y}^2 [C_y^2 + gC_x^2 (\frac{g}{4} + K)] \quad (1.10)$$

2. PROPOSED ESTIMATOR

Motivated by Sharma and Tailor (2010), the following exponential ratio-cum-dual to ratio estimator has been proposed to define as

$$t = \bar{y} \left[\alpha \exp\left(\frac{\bar{X} - \bar{x}}{\bar{X} + \bar{x}}\right) + (1 - \alpha) \exp\left(\frac{\bar{x}^* - \bar{X}}{\bar{x}^* + \bar{X}}\right) \right] \quad (2.1)$$

where α is a real constant to be determined such that the MSE of t is minimum. For $\alpha = 1$, t reduces to the estimator t_1 and for $\alpha = 0$, it reduces to the estimator t_2 . To obtain the bias and mean squared error (MSE) of the estimators, let

$\bar{y} = \bar{Y}(1 + e_0)$ and $\bar{x} = \bar{X}(1 + e_1)$ such that $E(e_i) = 0, i = 0, 1$ and $E(e_1^2) = f C_x^2$ and $E(e_0 e_1) = f C_{xy} = f \rho C_y C_x$

Expressing (2.1) in terms of e 's, we have

$$t = \bar{Y}(1 + e_0) \left[\alpha \exp\left(\frac{-e_1}{2}\right) + (1 - \alpha) \exp\left(\frac{ge_1}{2}\right) \right] \quad (2.2)$$

Expanding the right hand side of (2.2) and retaining terms up to second powers of e 's, and then subtracting \bar{Y} from both sides, we have

$$t - \bar{Y} = \bar{Y} \left[1 + e_0 + \alpha_1 \frac{e_1}{2} + \alpha_2 \frac{e_1^2}{8} + \alpha_1 \frac{e_0 e_1}{2} \right] - \bar{Y} \quad (2.3)$$

where $\alpha_1 = [g - (1 + g)\alpha]$ and $\alpha_2 = [g^2 + (1 - g^2)\alpha]$.

Taking expectations on both sides of (2.3), we get the bias of the estimator t up to the first order of approximation, as

$$B(t) = f \bar{Y} \left[\alpha_2 \frac{C_x^2}{8} + \frac{\alpha_1}{2} \rho C_y C_x \right] \quad (2.4)$$

From equation (2.3), we have

$$(t - \bar{Y}) \cong \bar{Y} (e_0 + \alpha_1 \frac{e_1}{2}) \quad (2.5)$$

Squaring both sides of equation (2.5) gives

$$(t - \bar{Y})^2 = \bar{Y}^2 \left[e_0^2 + \alpha_1^2 \frac{e_1^2}{4} + \alpha_1 e_0 e_1 \right] \quad (2.6)$$

and now taking expectation, we get the MSE of the estimator t , to the first order of approximation as

$$MSE(t) = f \bar{Y}^2 [C_y^2 + \alpha_1^2 \frac{C_x^2}{4} + \alpha_1 \rho C_y C_x] \quad (2.7)$$

which is minimum for optimum value of α as

$$\alpha = \frac{2K + g}{1 + g} = \alpha_{opt} \text{ (say)}$$

and the minimum MSE of t is

$$MSE_{\min}(t) = f \bar{Y}^2 C_y^2 (1 - \rho^2) = MSE(t)_{opt} \quad (2.8)$$

which is same as that of traditional linear regression estimator.

3. EFFICIENCY COMPARISON

We know that the variance of the sample mean \bar{y} is

$$V(\bar{y}) = f \bar{Y}^2 C_y^2 \quad (3.1)$$

Now we have from (3.1) and (2.8), that

$$V(\bar{y}) - MSE(t)_{opt} = \rho^2 \geq 0 \quad (3.2)$$

Showing that proposed estimator t is better than the per unit estimator of population mean.

From (1.2) and (2.8), we have

$$MSE(\bar{y}_R) - MSE(t)_{opt} = (C_x - \rho C_y)^2 \geq 0 \quad (3.3)$$

Which shows that proposed estimator is better than the traditional ratio estimator of Cochran (1940).

From (1.4) and (2.8), we have

$$MSE(\bar{y}_R^{(d)}) - MSE(t)_{opt} = (gC_x - \rho C_y)^2 \geq 0 \quad (3.4)$$

Thus t is better than the estimator $\bar{y}_R^{(d)}$ due to Srivenkataramana (1980).

From (1.6) and (2.8), we have that both the estimators are equally efficient.

From (1.8) and (2.8), we have

$$MSE(t_1) - MSE(t)_{opt} = \left(\frac{C_x}{2} - \rho C_y\right)^2 \geq 0 \quad (3.5)$$

Showing that proposed estimator t is better than the Bahl and Tuteja (1991) estimator t_1 .

From (1.10) and (2.8), we have

$$MSE(t_2) - MSE(t)_{opt} = \left(\frac{C_x}{2} + \rho C_y\right)^2 \geq 0 \quad (3.6)$$

Which shows that proposed estimator t is better than the estimator t_2 .

1. THE EMPIRICAL STUDY

We have used the data in Koyuncu and Kadilar (2009) to compare the efficiencies between the previous and the proposed estimator for the population mean under simple random sampling.

Data Statistics

Table no. 1

$N = 923$	$n = 180$	$\bar{Y} = 436.4345$	$\bar{X} = 11440.4984$
$C_y = 171833$	$C_x = 1.864528$	$\rho = 0.9543$	$g = 0.24226$

The percentage relative efficiencies of previously developed and proposed estimators with respect to usual unbiased estimator \bar{y} of population mean \bar{Y} have been computed and presented in table 2, below.

Percentage relative efficiencies of different estimator's w.r.t. \bar{y}

Table no.2

Estimator	Values of
\bar{y}	100.000
\bar{y}_R	939.649
$\bar{y}_R^{(d)}$	176.247
t_1	386.307
t_2	78.856
$(t)_{opt} = (\hat{Y}_{bkl})_{opt}$	1123.596

5. CONCLUSION

In the light of above numerical demonstration, we conclude that the proposed estimator t performs better than the usual estimator \bar{y} , Cochran (1940) usual ratio

estimator \bar{y}_R , Srivenkataramana (1980) dual to ratio estimator $\bar{y}_R^{(d)}$, Bahl and Tuteja (1991) exponential ratio type estimator and exponential dual to ratio type estimator. So the proposed estimator should be preferred over above estimators for the estimation of population mean of the study variable using auxiliary information.

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